e-content for students

B. Sc.(honours) Part 1paper 1

Subject:Mathematics

Topic:Transpose of a matrix

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Transpose of matrics

Definition. *The matrix obtained from any given matrix A, by interchanging its rows and columns is called the transpose of A and is denoted by A' or A^T.*

For example, if $A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \\ 6 & 8 \end{bmatrix}$, then $A' = \begin{bmatrix} 1 & 4 & 6 \\ 3 & 5 & 8 \end{bmatrix}$.

It follows that if A is a $m \times n$ matrix, then A' will be a $n \times m$ matrix and the (i, j)th element of A is equal to (j, i)th element of A'.

Thus if $A = [a_{ij}]$, then $A' = [a_{ji}]$.

properties

(i) $(A')' = A$.

Let $\quad A = [a_{ij}]$.

$\therefore \quad A' = [a_{ji}]$.

$\therefore \quad (A')' = [a_{ij}] = A$.

(ii) $(cA)' = cA'$ **where c is scalar.**

We have, $cA = c[a_{ij}] = [ca_{ij}]$.

$\therefore \quad (cA)' = [ca_{ji}] = c[a_{ji}] = cA'$.

(iii) $(A + B)' = A' + B'$ **where A and B are conformal for addition.**

Let $A = [a_{ij}]$ and $B = [b_{ij}]$.

$\therefore \quad A + B = [a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}]$.

$\therefore \quad (A + B)' = [a_{ji} + b_{ji}] = [a_{ji}] + [b_{ji}] = A' + B'$.

(iv) Reversal rule : $(AB)' = B'A'$ when A and B are conformal for product AB.

Let $A = [a_{ij}]$ and $B = [b_{jk}]$ be the $m \times n$ and $n \times p$ matrices respectively.

Then $A' = [a_{ji}]$ and $B' = [b_{kj}]$ will be the $n \times m$ and $p \times n$ matrices respectively.

Now $AB = [a_{ij}] \times [b_{jk}] = C$; where $c_{ik} = \sum\limits_{j=1}^{n} a_{ij} b_{jk}$ and it is a $m \times p$ matrix.

$$\therefore\ (AB)' = C' \text{ where } C_{ki} = \sum_{j=1}^{n} a_{ij}\, b_{jk} = \left[\sum_{j=1}^{n} b_{jk} a_{ij}\right];$$

where $i = 1, 2, 3, \ldots m; k = 1, 2, 3, \ldots p$ ...(1)

Now, the elements in the kth row of B' are the elements of the kth column of B.

They are $b_{1k}, b_{2k}, b_{3k}, \ldots b_{nk}$.

Similarly the elements of the ith column of A' are

$$a_{i1}, a_{i2}, a_{i3}, \ldots a_{in}.$$

The scalar product of these two sets of elements

$$= \sum_{j=1}^{n} b_{jk} a_{ij}$$

$$\therefore\quad B'A' = \left[\sum_{j=1}^{n} b_{jk} a_{ij}\right]; i = 1, 2, 3, \ldots m; k = 1, 2, 3, \ldots p \qquad \text{...(2)}$$

Therefore, from (1) and (2), we get $(AB)' = B'A'$.

symmetric &skew-symmetric matrices

Symmetric matrix : Definition : A square matrix $A = [a_{ij}]$ is said to be *symmetric* if $A = A'$ i.e., if $a_{ij} = a_{ji}$ i.e., the (i, j)th element is the same as the (j, i)th element.

Thus in a symmetric matrix $a_{ij} = a_{ji}$ for all i, j i.e., $a_{12} = a_{21}$, $a_{13} = a_{31}$, $a_{23} = a_{32}$, ...

For example, if $A = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ then $A' = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

$\therefore \quad A = A'$

$\therefore$ A is symmetric.

Skew-symmetric matrices : Definition : A square matrix $A = [a_{ij}]$ is said to be *skew-symmetric* if $A = -A'$ i.e., if $a_{ij} = -a_{ji}$ i.e. the (i, j)th element is the negative of the (j, i)th element for all i, j.

Since, by definition $a_{ii} = -a_{ii} \Rightarrow 2a_{ii} = 0 \;\therefore\; a_{ii} = 0$.

Therefore the diagonal elements of skew-symmetric matrix are always zero.

For example, the matrix $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ is skew-symmetric.